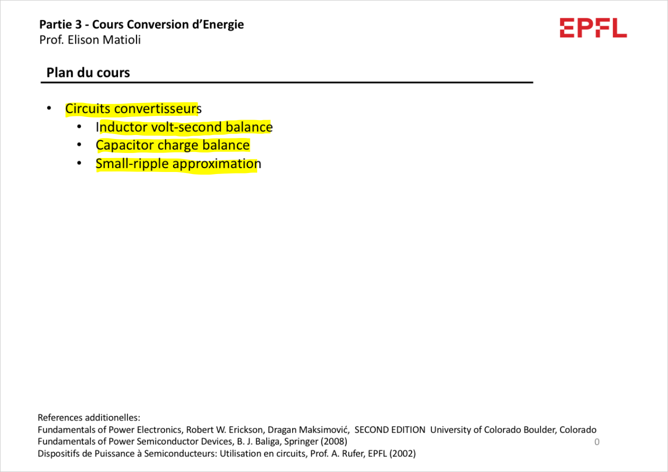
# Course 2

Monday, 25 November 2024 21:35

Cours 2 - convertiss...

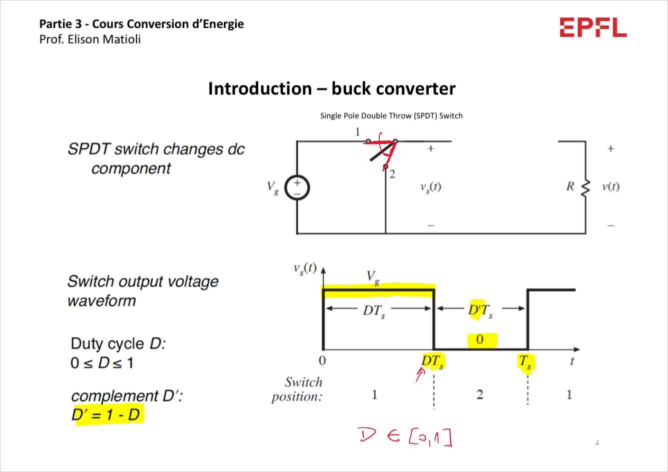


**Partie 3 - Cours Conversion d'Energie**
Prof. Elison Matioli

## Plan du cours

- Circuits convertisseurs
  - Inductor volt-second balance
  - Capacitor charge balance
  - Small-ripple approximation

References additionelles:
Fundamentals of Power Electronics, Robert W. Erickson, Dragan Maksimović, SECOND EDITION University of Colorado Boulder, Colorado
Fundamentals of Power Semiconductor Devices, B. J. Baliga, Springer (2008)
Dispositifs de Puissance à Semiconducteurs: Utilisation en circuits, Prof. A. Rufer, EPFL (2002)



**Partie 3 - Cours Conversion d'Energie**
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## Introduction – buck converter

Single Pole Double Throw (SPDT) Switch

*SPDT switch changes dc component*

*Switch output voltage waveform*

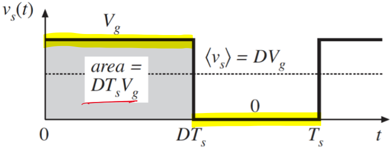
Duty cycle *D*:
$0 \leq D \leq 1$

*complement D′:*
*$D' = 1 - D$*

$D \in [0,1]$

## buck converter

— $V_s$ ⊓ $V_g$



*Fourier analysis: Dc component = average value*

$$\langle v_s \rangle = \frac{1}{T_s}\int_0^{T_s} v_s(t)\,dt = \frac{1}{T_s}\cdot\left(DT_s\cdot V_s + 0\right) = D\cdot V_g$$

$$\langle v_s \rangle = \frac{1}{T_s}(DT_sV_g) = DV_g$$

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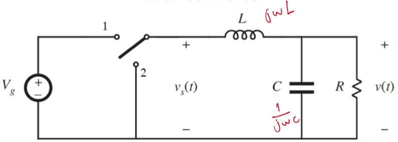
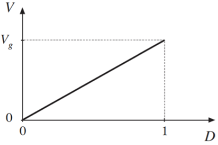
## buck converter



$v \approx \langle v_s \rangle = DV_g$



Filtre LC pour éliminer les harmoniques à haute fréquence

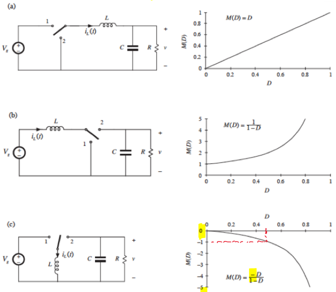
$M(D) = \frac{V_{out}}{V_{in}}$

## Convertisseurs DC/DC

*Buck*

*Boost*

*Buck-boost*



**Comment calculer le facteur M(D)?**

Introduire les principes de:

- Inductor volt-second balance
- Capacitor charge balance
- Small-ripple approximation

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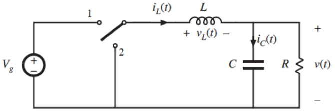
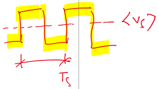
## Méthode: volt-second balance et capacitor-charge balance

*Buck converter containing practical low-pass filter*

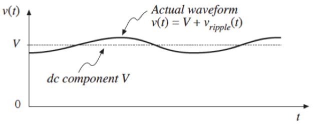




$f_s = 1/T_s$

*Actual output voltage waveform*

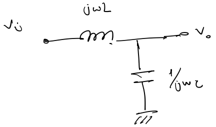
$v(t) = V + v_{ripple}(t)$



**Impossible de construire un filtre passe bas parfait**:

- Il y aura toujours des ondulations à la sortie… mais nous les considérons très faibles
- **Approximation de faibles ondulations** (small-ripple approximations)

①

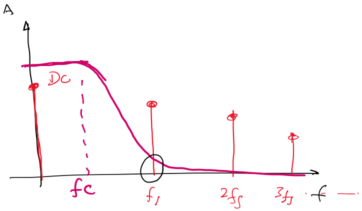


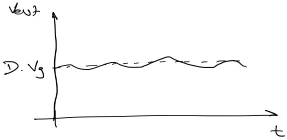
$$\frac{V_o}{V_i} = \frac{1/j\omega C}{1/j\omega C + j\omega L}$$

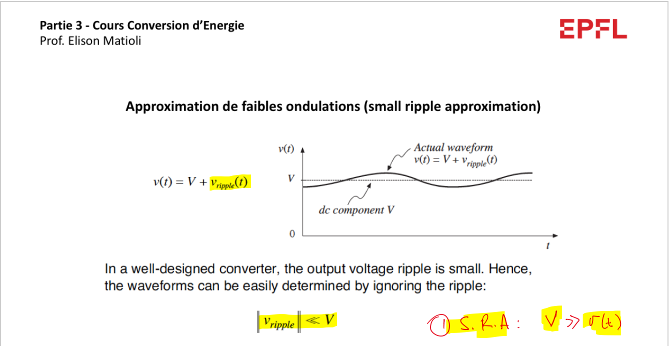
$$= \frac{1}{1 - \omega^2 LC}$$

$$\Rightarrow \boxed{f_c = \frac{1}{2\pi\sqrt{LC}}}$$

$$\boxed{f_c \leq f_s}$$







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EPFL

## Approximation de faibles ondulations (small ripple approximation)

$$v(t) = V + v_{ripple}(t)$$

v(t)
V
0
t
Actual waveform
$v(t) = V + v_{ripple}(t)$
dc component V

In a well-designed converter, the output voltage ripple is small. Hence, the waveforms can be easily determined by ignoring the ripple:

$$\left\| v_{ripple} \right\| \ll V$$

① S.R.A : $V \gg v(t)$

$$v(t) \approx V$$

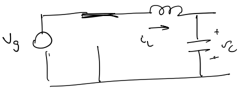
- On considère que la composante DC est beaucoup plus grande que l'AC: v(t) ~ V
- Cela élimine la dépendance des variables d'intérêt avec *t*

## Approximation de faibles ondulations (small ripple approximation)

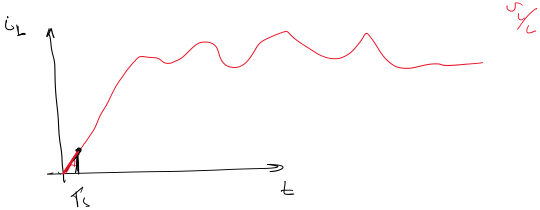
Approximation de **faibles ondulations** (small-ripple approximation) **est valable pour**:

- Des ondulations faibles
- Forme d'ondes continue $\left\{ \begin{matrix} i_L \\ v_C \end{matrix} \right.$
- **N'est pas valable pour des signaux discontinus** (provenus des commutations, par exemple $v_L$)
- Cette approximation est appliquée pour: **courant sur les inducteurs** ou **tension sur les condensateurs**
- La période $T_s$ est petite comparée aux constants de temps du circuit.

Cette approximation simplifie la solution, spécialement sur les filtres:
**Il n'y pas besoin de résoudre des équations différentielles**

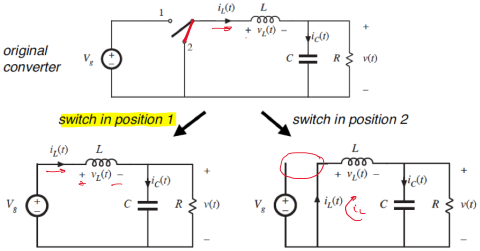


$$\left\{ \begin{aligned} v_L &= L\frac{di}{dt} \\ i_C &= C\frac{dv}{dt} \end{aligned} \right.$$



② ③

## Méthode: volt-second balance et capacitor charge balance
## Application: convertisseur "buck"



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## Méthode: volt-second balance et capacitor charge balance
## Application: convertisseur "buck"

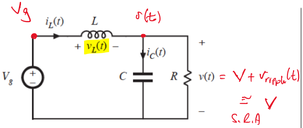
**Position 1**

*Inductor voltage*

$v_L = V_g - v(t)$

① *Small ripple approximation:*

$v_L \approx V_g - V$



*Knowing the inductor voltage, we can now find the inductor current via*

$$v_L(t) = L\frac{di_L(t)}{dt}$$

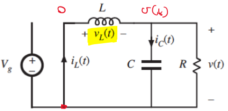
*Solve for the slope:*

$$\frac{di_L(t)}{dt} = \frac{v_L(t)}{L} \approx \frac{V_g - V}{L}$$

⇒ *The inductor current changes with an essentially constant slope*

## Méthode: volt-second balance et capacitor charge balance
## Application: convertisseur “buck”

**Position 2**



*Inductor voltage*

$$v_L(t) = -v(t)$$

① *Small ripple approximation:*

$$v_L(t) \approx -V$$

*Knowing the inductor voltage, we can again find the inductor current via*

$$v_L(t) = L\frac{di_L(t)}{dt}$$

*Solve for the slope:*

$$\frac{di_L(t)}{dt} \approx -\frac{V}{L}$$

Negative slope

⇒ *The inductor current changes with an essentially constant slope*

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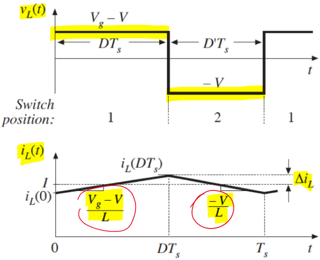
## Méthode: volt-second balance et capacitor charge balance
## Application: convertisseur “buck”

Plots de courant et tension



inductor current slope is constant

$$\langle v_L \rangle \Big|_{T_s} = 0$$

$$(v_g - v)\cdot D\cancel{T_s} + (-v)\cdot D'\cancel{T_s} = 0$$

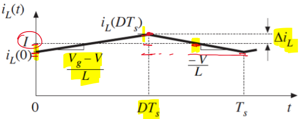
$$v_g D - v(\underbrace{\cancel{D + D'}}_{1}) = 0$$

$$v_L(t) = L\frac{di_L(t)}{dt}$$

$$\Rightarrow \boxed{v = D\cdot v_g}$$

## Méthode: volt-second balance et capacitor charge balance
## Application: convertisseur "buck"

**Ripple**



$= \frac{I - I_L(0)}{2}$

$$(\textit{change in } i_L) = (\textit{slope})(\textit{length of subinterval})$$

$$\left(2\Delta i_L\right) = \left(\frac{V_g - V}{L}\right)\left(DT_s\right)$$

$$\Rightarrow \quad \Delta i_L = \frac{V_g - V}{2L} DT_s \qquad \boxed{L = \frac{V_g - V}{2\Delta i_L} DT_s}$$

**value of inductance in the buck converter**

- Knowledge of the **peak current** is necessary when specifying the ratings of these devices
- Typical values of $\Delta i_L$ lie in the **range of 10% to 20%** of the full-load value of the dc component *I*
- It is undesirable to allow $\Delta i_L$ to become too large:
  - increase the peak currents of the inductor and of the semiconductor switching devices
  - increase the size and cost of the inductors
- The small-ripple approximation $I_L = I$ is usually justified for the inductor current
  - The problem could be solved exactly but results are typically very close to this approximation.

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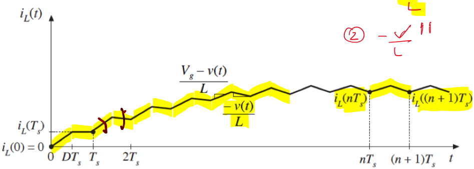
## Méthode: volt-second balance et capacitor charge balance
## Application: convertisseur "buck"

**Régime de transitoire (turn on)**

① $\frac{V_g - V^{qq}}{L}$

② $-\frac{V^{tt}}{L}$



When the converter operates in equilibrium:

$$i_L((n+1)T_s) = i_L(nT_s)$$

## Principe "volt-second balance"

Inductor defining relation:

$$\int_0^{T_s} v_L(t) = L\int_0^{T_s}\frac{di_L(t)}{dt}$$

Integrate over one complete switching period:

$$i_L(T_s) - i_L(0) = \frac{1}{L}\int_0^{T_s} v_L(t)\,dt = 0$$

② $\langle v_L\rangle\big|_{T_s} = 0$

In periodic steady state, the net change in inductor current is zero:

$$0 = \int_0^{T_s} v_L(t)\,dt$$

*Hence, the total area (or volt-seconds) under the inductor voltage waveform is zero whenever the converter operates in steady state.*

An equivalent form:

$$0 = \frac{1}{T_s}\int_0^{T_s} v_L(t)\,dt = \langle v_L\rangle$$

*The average inductor voltage is zero in steady state.*

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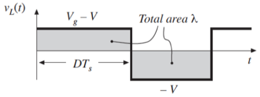
## Principe "volt-second balance": convertisseur "buck"

*Inductor voltage waveform, previously derived:*



Integral of voltage waveform is area of rectangles:

$$\lambda = \int_0^{T_s} v_L(t)\,dt = (V_g - V)(DT_s) + (-V)(D'T_s)$$

Average voltage is

$$\langle v_L\rangle = \frac{\lambda}{T_s} = D(V_g - V) + D'(-V)$$

Equate to zero and solve for *V*:

$$0 = DV_g - (D + D')V = DV_g - V \quad \Rightarrow \quad \boxed{V = DV_g}$$

$M(D) = D \Rightarrow$ buck!

**Principe inducteur volt-second balance:**

- Nous permet de dériver facilement l'expression DC de la tension de sortie
- Le même résultat, sans utiliser l'argument de transformé de Fourier pour la composante DC
- C'est un principe général pour tous les types de convertisseurs

③

## Principe "capacitor-charge balance": convertisseur "buck"

Capacitor defining relation:

$$i_C(t) = C\frac{dv_C(t)}{dt}$$

Integrate over one complete switching period:

$$v_C(T_s) - v_C(0) = \frac{1}{C}\int_0^{T_s} i_C(t)\,dt$$

In periodic steady state, the net change in capacitor voltage is zero:

$$0 = \frac{1}{T_s}\int_0^{T_s} i_C(t)\,dt = \langle i_C \rangle$$

*Hence, the total area (or charge) under the capacitor current waveform is zero whenever the converter operates in steady state. The average capacitor current is then zero.*

**Intuitive result:**

- If a dc current is applied to a capacitor:
  then the capacitor will charge continually and its voltage will increase without bound
- if a dc voltage is applied to an inductor:
  then the flux will increase continually and the inductor current will increase without bound

**Principle of capacitor charge balance: steady-state currents in a switching converter**

3 principes :

① S.R.A : $v(t) = V + v_{ripple}(t) \simeq V$

② $\langle v_L \rangle \Big|_{T_s} = 0$

③ $\langle i_c \rangle \Big|_{T_s} = 0$

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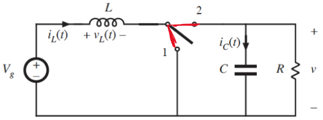
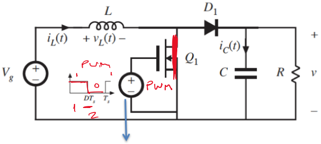
## Exercise: convertisseur boost

*Boost converter with ideal switch*



*Realization using power MOSFET and diode*



Gate driver:
Position1: driver turns on the MOSFET
Position2: driver turns off the MOSFET

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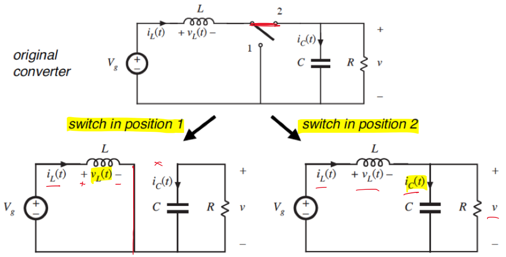
## Exercise: convertisseur boost



Ecrivez l'orientation des tensions et courants avant de commencer la résolution

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## Exercise: convertisseur boost

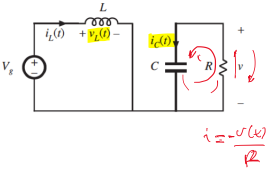
**Position 1**

*Inductor voltage and capacitor current*

$$v_L = V_g$$
$$i_C = -v / R$$

① *Small ripple approximation:*

$$v_L = V_g$$
$$i_C = -V / R$$



$i = -\frac{v(t)}{R}$

## Exercise: convertisseur boost

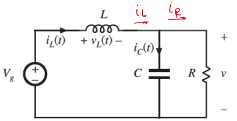
### Position 2

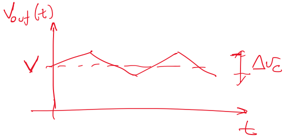
*Inductor voltage and capacitor current*

$$v_L = V_g - v$$
$$i_C = i_L - v / R$$

① *Small ripple approximation:*

$$v_L = V_g - V$$
$$i_C = I - V / R$$





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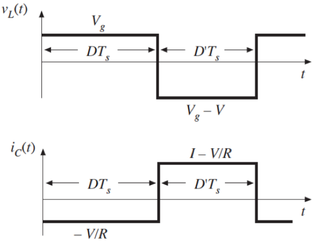
## Exercise: convertisseur boost

### Tension et courant



②
$$\langle v_L\rangle\Big|_{T_s} = 0$$

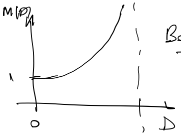
③
$$\langle i_C\rangle\Big|_{T_s} = 0$$

$$\langle v_L\rangle\Big|_{T_s} = \frac{V_g \cdot D\,\cancel{T_s} + (V_g - V)\,D'\,\cancel{T_s}}{\cancel{T_s}} = 0$$

$$V_g D + V_g D' - V D' = 0$$

$$V = \frac{V_g \overset{1}{\cancel{(D+D')}}}{D'} = \frac{V_g}{D'} = \frac{V_g}{1-D}$$

$$M(D) = \frac{V}{V_g} = \frac{1}{1-D}$$

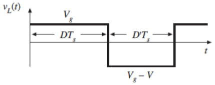


Boost

## Exercise: convertisseur boost

### Calcul de la tension DC de sortie V:

Net volt-seconds applied to inductor over one switching period:



$$\int_0^{T_s} v_L(t)\, dt = (V_g)\, DT_s + (V_g - V)\, D'T_s$$

$$V_g \cdot D \cancel{T_s} + (V_g - V) \cdot D' \cancel{T_s} = 0$$

Equate to zero and collect terms:

$$V_g\,(D + D') - V\,D' = 0$$

$$V_g(D + D') - V \cdot D' = 0$$

Solve for V:

$$V = \frac{V_g}{D'}$$

The voltage conversion ratio is therefore

$$M(D) = \frac{V}{V_g} = \frac{1}{D'} = \frac{1}{1-D}$$

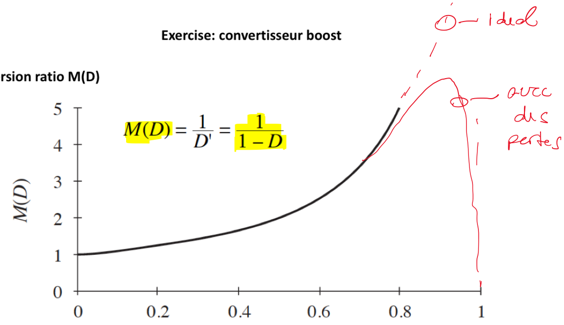
$$M(D) = \frac{1}{1-D}$$

boost !

## Exercise: convertisseur boost

### Conversion ratio M(D)

$$M(D) = \frac{1}{D'} = \frac{1}{1-D}$$



D

↑

PWM

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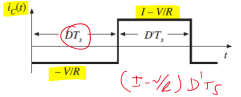
### Exercise: convertisseur boost

$-\frac{V}{R}\,D\cancel{T_s} + (I-\frac{V}{R})\,D'\cancel{T_s} = 0$

### Calcul du courant DC sur l'inductance L



Capacitor charge balance:

$$\int_0^{T_s} i_C(t)\,dt = \underline{(-\frac{V}{R})\,DT_s} + \underline{(I-\frac{V}{R})\,D'T_s}$$

Collect terms and equate to zero:

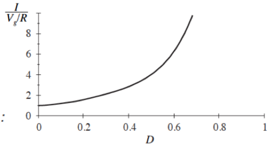
$$-\frac{V}{R}(D+D') + I\,D' = \underline{0}$$

Solve for $I$:

$$I = \frac{V}{D'R}$$

Eliminate $V$ to express in terms of $V_g$:

$$I = \frac{V_g}{D'^2 R}$$



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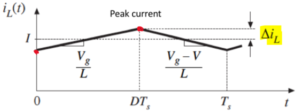
### Exercise: convertisseur boost

### Calcul du courant "ripple" de l'inducteur



Inductor current slope during subinterval 1:

$$\frac{di_L(t)}{dt} = \frac{v_L(t)}{L} = \frac{V_g}{L}$$

Inductor current slope during subinterval 2:

$$\frac{di_L(t)}{dt} = \frac{v_L(t)}{L} = \frac{V_g - V}{L}$$

Change in inductor current during subinterval 1 is *(slope) (length of subinterval)*:

$$2\Delta i_L = \frac{V_g}{L}\,DT_s$$

Solve for peak ripple:

$$\boxed{\Delta i_L = \frac{V_g}{2L}\,DT_s}$$

- Choose $L$ such that desired ripple magnitude is obtained

## Exercise: convertisseur boost

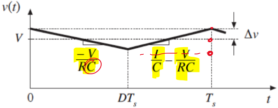
### Calcul de la tension "ripple" du condensateur

Capacitor voltage slope during subinterval 1:

$$\frac{dv_C(t)}{dt} = \frac{i_C(t)}{C} = \frac{-V}{RC}$$

Capacitor voltage slope during subinterval 2:

$$\frac{dv_C(t)}{dt} = \frac{i_C(t)}{C} = \frac{I}{C} - \frac{V}{RC}$$



Change in capacitor voltage during subinterval 1 is *(slope) (length of subinterval)*:

$$-2\Delta v = \frac{-V}{RC} DT_s$$

Solve for peak ripple:

$$\Delta v = \frac{V}{2RC} DT_s$$

- Choose *C* such that desired voltage ripple magnitude is obtained
- In practice, capacitor *equivalent series resistance* (esr) leads to increased voltage ripple